Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 3

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory.
2. Section - A carries 20 marks weightage, Section - B carries 10 marks weightage, Section - C carries 18 marks weightage, Section - D carries 20 marks weightage and Section - E carries 3 case-based with total weightage of 12 marks.
3. **Section – A:** It comprises of 20 MCQs of 1 mark each.
4. **Section – B:** It comprises of 5 VSA type questions of 2 marks each.
5. **Section – C:** It comprises of 6 SA type of questions of 3 marks each.
6. **Section – D:** It comprises of 4 LA type of questions of 5 marks each.
7. **Section – E:** It has 3 case studies. Each case study comprises of 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in each case-study.
8. Internal choice is provided in 2 questions in Section - B, 2 questions in Section – C, 2 questions in Section - D. You have to attempt only one of the alternatives in all such questions.

Section A

1. If A is a square matrix of order 3 such that A(adj A) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$, then |A| is equal to **[1]**

a) -6 b) -3

c) 9 d) 3

2. A sampling distribution might be based on which of the following? **[1]**

a) Sample proportions b) Sample correlations

c) All of these d) Sample means

3. The declared rate of return compounded semiannually which is equivalent to 10.25%, effective rate of return, is: **[1]**

a) 9.89% b) 10.13%

c) 10% d) 10.05%

4. Region represented by x $\geq$ 0, y $\geq$ 0 lies in **[1]**

a) IV quadrant b) I quadrant

c) II quadrant d) III quadrant

5. The equation of tangent to the curve y (1 + x^2) = 2 - x, where it crosses x-axis is: **[1]**

a) 5x - y = 2
b) x - 5y = 2
c) x + 5y = 2
d) 5x + y = 2

6. If the mean and variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than 1 is **[1]**

a) $\frac{4}{5}$
b) $\frac{15}{16}$
c) $\frac{7}{8}$
d) $\frac{2}{3}$

7. A bag contains 2 white and 4 black balls. A ball is drawn 5 times with replacement. The probability that atleast 4 of the balls drawn are white is **[1]**

a) $\frac{10}{243}$
b) $\frac{32}{243}$
c) $\frac{11}{243}$
d) $\frac{8}{243}$

8. Solution of the differential equation $x\frac{dy}{dx} + 2y$ = x^2 is **[1]**

a) $y = \frac{x^2+C}{4x^2}$
b) $y = \frac{x^2+C}{x^2}$
c) $y = \frac{x^2}{4} + \text{C}$
d) $y = \frac{x^4+\text{C}}{4x^2}$

9. A pipe fills $\frac{3}{7}^{\text{th}}$ part of a tank in 1 hour. The rest of the tank can be filled in **[1]**

a) $\frac{4}{3}$ hours
b) $\frac{7}{3}$ hours
c) $\frac{7}{4}$ hours
d) $\frac{3}{4}$ hours

10. Find a second order differential equation: **[1]**

a) y''' + $(y'')^2$ + y = 0
b) y'y'' + y = x^2
c) y' = y^2
d) $(y')^2$ + x = y^2

11. In what ratio must a grocer mix two varieties of pulses costing ₹ 85 per kg and ₹ 100 per kg respectively so as to get a mixture worth ₹ 92 per kg? **[1]**

a) 8 : 7
b) 5 : 7
c) 7 : 5
d) 7 : 8

12. If $\frac{1}{2}\left(\frac{3}{5}x + 4\right) \geq \frac{1}{3}(x - 6)$, x ∈ R, then **[1]**

a) x ∈ $(-\infty, 120)$
b) x ∈ $[120, \infty)$
c) x ∈ $(-\infty, 120]$
d) x ∈ $(120, \infty)$

13. Two pipes A and B can fill a cistern in 36 and 48 minutes respectively. Both pipes are opened together, after how many minutes should B be turned off, so that the cistern be filled in 24 minutes? **[1]**

a) 12 minute
b) 10 minute
c) 6 minute
d) 16 minute

14. The maximum value of Z = 4x + 2y subjected to the constraints 2x + 3y $\leq$ 18, x + y $\geq$ 10; x, y $\geq$ 0 is **[1]**

a) 20
b) 40
c) 36
d) none of these

15. If x and a are real numbers such that a > 0 and $|x|$ > a, then **[1]**

a) x ∈ [$-\infty$, a]

b) $x \in (-\infty, -a) \cup (a, \infty)$

c) x ∈ (-a, ∞)

d) x ∈ (-a, a)

16. A specific characteristic of a population is known as a **[1]**

a) mean

b) statistic

c) a sample

d) parameter

17. $\int \frac{1}{x+x\log x}$ dx is equal to: **[1]**

a) 1 + log x + C

b) log (1 + log x) + C

c) x log (1 + log x) + C

d) x + log x + C

18. In the measurement of the secular trend, the moving averages: **[1]**

a) Measure the seasonal Variations

b) Measure the time Variations

c) Smooth out the time series

d) Given the trend in a straight line

19. **Assertion (A):** If A is a square matrix of order 3 such that |adj A| = 144, then the value of |A| is $\pm$12. **[1]**

Reason (R): If A is an invertible matrix of order n, then |adj A| = $|A|^{n-1}$.

a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).

c) Assertion (A) is true but Reason (R) is false.

d) Assertion (A) is false but Reason (R) is true.

20. **Assertion (A):** The function f(x) = x^x, x > 0 is strictly increasing in $\left[\frac{1}{e}, \infty\right)$. **[1]**

Reason (R): $\log_a x > b \Rightarrow x > a^b$ if a > 1.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Calculate the 3-yearly moving averages of the following data: **[2]**

Year	1	2	3	4	5	6	7
Value	2	4	5	7	8	10	13

22. Sanjay takes a personal loan of ₹500000 at the rate of 12% per annum for 3 years. Calculate his EMI by using flat rate method. **[2]**

OR

At what rate per cent, per annum compounded annually, will the sum of money become 4 times of itself in 2 years?

23. Evaluate: **[2]**

$$\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$$

24. A company establishes a sinking fund to provide for the payment of ₹1,00,000 debt, maturing in 4 years. Contributions to the fund are to be made at the end of year. Find the amount of each annual deposit if interest is 18% per annum. [Use $(1.18)^4$ = 1.9388] **[2]**

OR

Rahul purchased an old scooter for ₹ 16000. If the cost of the scooter after 2 years depreciates to ₹14440, find the rate of depreciation.

25. A bottle is full of Dettol. One-third of its is taken out and then an equal amount of water is poured into the bottle to fill it. This operation is repeated four times. Find the final ratio of Dettol and water in the bottle. **[2]**

Section C

26. Solve the initial value problem: $e^{\frac{dy}{dx}} = x + 1$; y(0) = 3 **[3]**

OR

It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal.

i. If the interest is compounded continuously at 5% per annum, in how many years will ₹ 100 double itself?
ii. At what interest rate will ₹ 100 double itself in 10 years ?($\log_e 2 = 0.6931$)
iii. How much will ₹ 1000 be worth at 5% interest after 10 years? ($e^{0.5} = 1.648$)

27. A ₹2,000, 8% bond is redeemable at the end of 10 years at ₹105. Find the purchase price to yield 10% effective rate. **[3]**

28. The marginal cost function of a product is given by MC = $\frac{x}{\sqrt{x^2+400}}$. Find the total cost and the average cost if the fixed cost is ₹ 1000. **[3]**

29. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is What is the probability that he will win a prize $\frac{1}{100}$. **[3]**

i. at least once
ii. exactly once
iii. at least twice?

OR

In a normal distribution 31% of the articles are under 45 and 8% are over 64. Calculate the mean and standard deviation of the distribution.

30. Construct 5-year Moving averages from the following data of the number of industrial failure in a country during 2003-2018: **[3]**

Year	**No. of Failures**	**Year**	**No. of Failure**
2003	23	2011	9
2004	26	2012	13
2005	28	2013	11
2006	32	2014	14
2007	20	2015	12
2008	12	2016	9
2009	12	2017	3
2010	10	2018	1

31. Consider the following hypothesis test: **[3]**

$H_0 : \mu = 15$

$H_a : \mu \neq 15$.

A sample of 50 provided a sample mean of 14.15. The population standard deviation is 3.

i. Compute the value of the test statistic.

ii. What is the p-value?

iii. At $\alpha = 0.05$, what is your conclusion?

iv. What is the rejection rule using the critical value? What is your conclusion?

Section D

32. By using determinants, solve the following system of linear equations: **[5]**

$x + y + z = 1$

$x + 2y + 3z = 4$

$x + 3y + 5z = 7$

OR

The equilibrium conditions for three competitive markets are described as given below, where P_1, P_2 and P_3 are the equilibrium price for each market respectively: $P_1 + 2P_2 + 3P_3 = 85$, $3P_1 + 2P_2 + 2P_3 = 105$, $2P_1 + 3P_2 + 2P_3 = 110$

Using matrix method, find the values of respective equilibrium prices.

33. In a 1000-metre race, A, B and C get Gold, Silver and Bronze medals respectively. If A beats B by 100 metres and B beats C by 100 metres, then by how many metres does A beat C? **[5]**

34. An unbiased die is thrown twice. Find the probability distribution of the number of sixes. **[5]**

OR

Suppose 220 misprints are distributed randomly throughout a book of 200 pages. Find the probability that a given page contains

i. no misprints,

ii. one misprint,

iii. 2 misprints,

iv. 2 or more misprints.

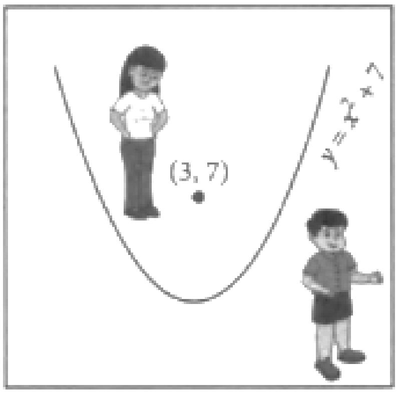
(Given $e^{-1.1} = 0.33287$)

35. A machine costing ₹50,000 depreciates at a constant rate of 8%. What is the depreciation charge for the 8th year. If the estimated useful life of the machine is 10 years, determine its scrap value. **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

A student Mohit is running on a playground along the curve given by $y = x^2 + 7$. Another student Preeti standing at point (3, 7) on playground wants to hit Mohit by paper ball when Mohit is nearest to Preeti.



(a) What will be the Mohit position at any value of x?

(b) What will be the distance (say D) between Mohit and Preeti?

(c) For which real value(s) of x, first derivative of D^2 w.r.t. **x** will Vanish?

OR

Find the position of Mohit when Preeti will hit the paper hall.

37. **Read the text carefully and answer the questions:** **[4]**

The nominal rate of return shows the yield of an investment over time without accounting for negative elements such as inflation or taxes. By calculating the nominal rate of return, you can compare the performance of your assets easily, regardless of the inflation rate or differing spans of time for each investment. By obtaining a bird's-eye view of how your assets are growing, you can make more prudent investment decisions in the future.

(a) A man invests a sum of money in ₹100 shares paying 15% dividend quoted at 20% premium. If his annual dividend is ₹540, calculate the rate of return on his investment.

(b) Mr. Satya holds 1500, ₹100 shares of a company paying 15% dividend annually quoted at 30% premium. Calculate rate of return on his investment.

(c) ₹100 shares of a company are sold at a discount of ₹ 20. If the return on the investment is 15%, find the rate of dividend declared.

OR

A company declared a dividend of 14%. Find the market value of ₹50 shares, if the return on the investment was 10%.

38. Aeroplane is an important invention for three reasons. It shortens travel time, is more comfortable and facilitates the transport of heavy cargo. **[4]**

An aeroplane can carry a maximum of 200 passengers.

A profit of ₹400 is made on each executive class ticket and a profit of ₹300 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However at least 4 times as many passenger prefer to travel by economy class than by executive class.

Based on above information answer the following questions.

i. If x tickets of executive class and y tickets of economy class be sold, then write the constraint.

ii. Write the correct pair of constraints.

iii. If profit earned by airlines is represented by Z, then write the constraint.

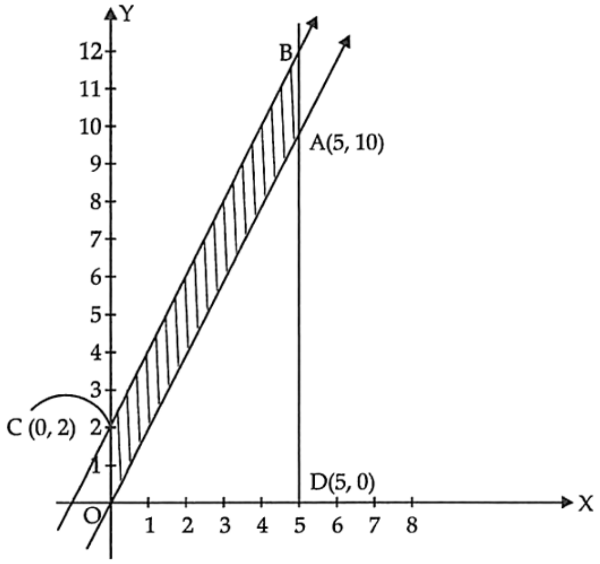
iv. Airlines are interested to maximise the profit. For this what are the value of x and y i.e.r number of executive class ticket and economy class ticket to be sold?

v. What is the maximum profit earned by airlines?

OR

Read the following text carefully and answer the questions that follow:

The feasible region for an LPP is shown in the following figure. The CB is parallel to OA.



i. What is the equation of line OA? (1)

ii. What is the equation of line BC? (1)

iii. What is the co-ordinates of point B? (2)

OR

What are the constraints for the L.P.P.? (2)

Solution

Section A

1.

(b) -3

Explanation:

$\because$ A(adj A) = |A| $I_3 \Rightarrow$ |A| = -3

2.

(c) All of these

Explanation:

All of these

3.

(c) 10%

Explanation:

10%

4.

(b) I quadrant

Explanation:

I quadrant

5.

(c) x + 5y = 2

Explanation:

We have, equation of the curve $y(1 + x^2) = 2 - x$ ...(i)

It is given that the curve crosses x-axis

Putting y = 0 in equation (i), we get

$\therefore 0(1 + x^2) = 2 - x$

$\Rightarrow$ x = 2

So the curves passes through the point (2, 0)

Now differentiating equation (i) w.r.t. x, we get

$\therefore y \times (0 + 2x) + \left(1 + x^2\right) \cdot \frac{dy}{dx}$ = 0 - 1

$\Rightarrow \frac{dy}{dx} = \frac{-1-2xy}{1+x^2}$

$\therefore \left(\frac{dy}{dx}\right)_{(2,0)} = \frac{-1-2\times 0}{1+2^2} = -\frac{1}{5}$ = slope of tangent to the curve

$\therefore$ Equation of tangent of the curve passing through (2, 0) is

y - 0 = $-\frac{1}{5}$ (x - 2)

or x + 5y = 2

6.

(b) $\frac{15}{16}$

Explanation:

Given that mean = 2, and variance = 1

np = 2, npq = 1

2(q) = 1

q = $\frac{1}{2}$.

Hence, p = $\frac{1}{2}$

n($\frac{1}{2}$) = 2

n = 4

The probability that X takes a value greater than 1 = P(X > 1)

P(X $\geq$ 1) = 1 - P(X = 0)

P(X $\geq$ 1) = 1 - $^4C_0$ $(\frac{1}{2})^0(\frac{1}{2})^{4-0}$

P(X $\geq$ 1) = 1 - [(1)(1)$(\frac{1}{2})^4$]

P(X $\geq$ 1) = 1 - ($\frac{1}{16}$)

P(X $\geq$ 1)= $\frac{15}{16}$

7.

(c) $\frac{11}{243}$

Explanation:

Total number of balls = 2 + 4 = 6

$p=\frac{2}{6}=\frac{1}{3} \Rightarrow q=1-\frac{1}{3}=\frac{2}{3}, n=5$

P(X $\geq$ 4) = P(X = 4) + P(X = 5)

$={}^5C_4\left(\frac{1}{3}\right)^4\frac{2}{3}+{}^5C_5\left(\frac{1}{3}\right)^5=\frac{10}{243}+\frac{1}{243}=\frac{11}{243}$

8.

(d) $y=\frac{x^4+C}{4x^2}$

Explanation:

$\frac{dy}{dx}+\frac{2}{x}$y = x $\Rightarrow$ I.F. = $e^{\int\frac{2}{x}dx}$ = $e^{2\log x}$ = x^2

$\therefore$ Solution is y $\cdot$ x^2 = $\int$ x $\cdot$ x^2 dx + C_1

y $\cdot$ x^2 = $\frac{x^4}{4}$ + C_1 $\Rightarrow$ y = $\frac{x^4+C}{4x^2}$

9. **(a)** $\frac{4}{3}$ hours

Explanation:

$\frac{3}{7}$l $\rightarrow$ 1 min

remaining amount = $1-\frac{3}{7}$ = $\frac{4}{7}$ltr

since time taken to fill 1 litre

= $\frac{3}{7}$min

So time required to fill $\frac{4}{7}$litres

= $\frac{7}{3}\times\frac{4}{7}$ = $\frac{4}{3}$min

10.

(b) y'y'' + y = x^2

Explanation:

y'y'' + y = x^2 is a second order differential equation.

11. **(a)** 8 : 7

Explanation:

Ratio $=\frac{100-92}{92-85}=\frac{8}{7}$ i.e. 8 : 7

12.

(c) x $\in$ $(-\infty$, 120]

Explanation:

$\frac{1}{2}\left(\frac{3}{5}x+4\right)\geq\frac{1}{3}$ (x - 6), x $\in$ R

$\Rightarrow$ $\frac{3}{10}$ x + 2 $\geq$ $\frac{1}{3}$ (x - 6)

Multiplying both sides by 30, we get

$9x + 60 \geq 10x - 60$

$\Rightarrow 60 + 60 \geq 10x - 9x \Rightarrow 120 \geq x$

$\Rightarrow x \leq 120$

$\therefore x \in (-\infty, 120]$

13.

(d) 16 minute

Explanation:

P can fill the cistern in 36 minutes, so in 1 min,

P can fill the cistern $= \frac{1}{36}$ th part

In 24 min, P can fill the cistern $= \frac{24}{36} = \frac{2}{3}$ rd.

Remaining part $= 1 - \frac{2}{3} = \frac{1}{3}$ rd

As Q can fill full cistern in 48 minutes, so it will fill $\frac{1}{3}$rd part in 16 minutes.

14.

(d) none of these

Explanation:

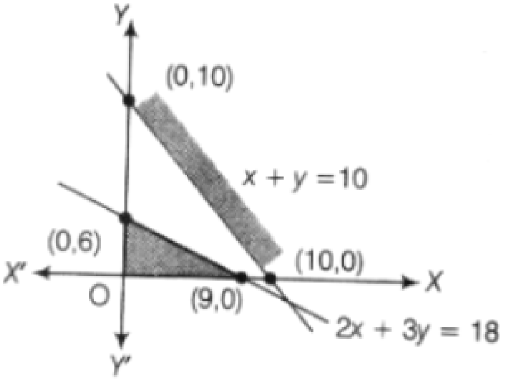
Z = 4x + 2y

Subject to constraints

$2x + 3y \leq 18$,

$x + y \geq$ and

$x, y \geq 0$



There is no common area in the first quadrant. Hence, the objective function Z cannot be maximized.

15.

(b) $x \in (-\infty, -a) \cup (a, \infty)$

Explanation:

$|x| > a$

$\Rightarrow x < -a$ or $x > a$

$\Rightarrow x \in (-\infty, -a) \cup (a, \infty)$

16.

(d) parameter

Explanation:

parameter

17.

(b) log (1 + log x) + C

Explanation:

Given the function

$\Rightarrow \int \frac{1}{(x+x\log x)} \cdot dx$

$\Rightarrow \frac{1}{x} \int \frac{1}{1+\log x)} \cdot dx$
let the $1+\log x = t$
$o + \frac{1}{x} \cdot dx = dt$
$\Rightarrow \frac{1}{x} \cdot dx = dt$
Hence
$\Rightarrow \int \frac{1}{x(1+\log x)} \cdot dx$
$\Rightarrow \int \frac{1}{t} \cdot dt$
$\Rightarrow \log t + C$
putting value of t
$\Rightarrow [\log[1+\log x] + C]$

18.

(c) Smooth out the time series

Explanation:

Moving averages is a series of arithmetic means of variate values of a sequence. This is another way of drawing a smooth curve for a time series data.

19.

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).

Explanation:

Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

$f(x) = x^x$
Taking logarithm on both sides, we get
$\log f(x) = x \log x$
Differentiating w.r.t.x, we get
$\frac{1}{f(x)} . f'(x) = \frac{x}{x} + \log x$
$\Rightarrow f'(x) = x^x (1 + \log x)$
For function to be strictly increasing $f'(x) > 0$
i.e. $x^x (1 + \log x) > 0 \Rightarrow 1 + \log x > 0$ ($\because x^x > 0$)
$\Rightarrow \log x > -1 \Rightarrow x > e^{-1}$ ($\because e > 1$)
$\Rightarrow x > \frac{1}{e}$
$\therefore$ f(x) is strictly increasing in $\left[\frac{1}{e}, \infty\right)$
$\therefore$ Assertion is true.
Also, $\log, x > b \Rightarrow x > a^b$ if $a > 1$ is true
$\therefore$ Reason is true.
Hence, Assertion and Reason both are true and Reason is the correct explanation of Assertion.

Section B

21.

Calculation of 3-year moving averages:

Year	Value	3-year moving total	3-year moving average
1	2		
2	4	11	3.667
3	5	16	5.333
4	7	20	6.667
5	8	25	8.333
6	10	31	10.333
7	13		

$\frac{1}{3}$

22. Given P = ₹500000, $i = \frac{12}{2 \times 100} = 0.01$
and $n = 12 \times 3 = 36$.

so, EMI = $\frac{P+Pni}{n}$ = $\frac{500000+500000\times36\times0.01}{36}$

= $\frac{500000+180000}{36}$ = $\frac{680000}{36}$

= 18888.89

Hence, EMI = ₹18888.89

OR

Interest for 1 year = ₹(4320 - 4000) = ₹ 320

Let rate of interest be r%

$\therefore \frac{4000\times r\times1}{100}$ = 320 ⇒ r = 8

∴ Rate of interest = 8%

∴ Amount after 3 years = 4000 $\left(1+\frac{8}{100}\right)^3$ = 4000(1.08)^3 = 4000 × 1.259 = ₹ 5036

23. Let I = $\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}dx$ ...(1)

Then, by using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$, we get

I = $\int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{a-(a-x)}}dx=\int_0^a \frac{\sqrt{a-x}}{\sqrt{a-x}+\sqrt{x}}dx$ ...(2)

On adding (1) and (2), we get

2I = $\int_0^a \frac{\sqrt{x}+\sqrt{a-x}}{\sqrt{x}+\sqrt{a-x}}dx=\int_0^a 1\,dx=[x]_0^a$ = a - 0 = a

⇒ I = $\frac{1}{2}a$

24. Let each annual deposit to the sinking fund be ₹R. Then R is given by

A = $RS_{\lceil ni}=R\left|\frac{(1+i)^n-1}{i}\right|$

$=R\left|\frac{(1.18)^4-1}{0.18}\right|$

$=R\left|\frac{1.9388-1}{0.18}\right|$

$=R\left|\frac{0.9388}{0.18}\right|$ = R(5.2156)

⇒ R = $\frac{1,00,000}{5.2156}$ = ₹19,173.25

OR

The current cost of the scooter, C_0 = 16000

Cost after two years, C = 14440

Let the rate of depreciation be R, then

C = $C_0\left(1-\frac{R}{100}\right)^T$

⇒ 14400 = 16000$\left(1-\frac{R}{100}\right)^2$

$\Rightarrow \frac{14400}{16000}=\left(1-\frac{R}{100}\right)^2$

$\Rightarrow \left(\frac{38}{40}\right)^2=\left(1-\frac{R}{100}\right)^2$

$\Rightarrow \frac{38}{40}=1-\frac{R}{100}$

$\Rightarrow \frac{R}{100}$ = 1 - $\frac{38}{40}$

⇒ R = $\frac{2\times100}{40}$

⇒ R = 5%

25. Let the original quantity of dettol be x litres and let y litres be taken out which is replaced by an equal quantity of water. It is given that y = $\frac{x}{3}$

After 4 operations the quantity of dettol left = $x\left(1-\frac{y}{x}\right)^4$

After 4 operations the quantity of water in the bottle = $x-x\left(1-\frac{y}{x}\right)^4$

∴ Thus, after 4 operations, we obtain

$\frac{\text{Quantity of dettol in the bottle}}{\text{Quantity of water in the bottle}}=\frac{x\left(1-\frac{y}{x}\right)^4}{x-x\left(1-\frac{y}{x}\right)^4}=\frac{x\left(1-\frac{x}{3x}\right)^4}{x-x\left(1-\frac{x}{3x}\right)^4}$ ... [∵ y = $\frac{x}{3}$]

$= \frac{\left(1-\frac{1}{3}\right)^4}{1-\left(1-\frac{1}{3}\right)^4} = \frac{16/81}{1-\frac{16}{81}} = \frac{16}{65}$

Hence, after four operations, dettol and water in the bottle are in two ratio 16 : 65.

Section C

26. The given differential equation is,

$e^{\frac{dy}{dx}} = x + 1$

Taking log on both sides, we get,

$\frac{dy}{dx} \log e = \log (x + 1)$

$\Rightarrow \frac{dy}{dx} = \log (x + 1)$

$\Rightarrow dy = \{\log (x + 1)\} dx$

Integrating both sides, we get

$\int dy = \int \{\log (x + 1) dx$

$\Rightarrow y = \int \underset{II}{1} \times \log(\underset{I}{x} + 1)dx$

$\Rightarrow y = \log (x + 1) \int 1\, dx - \int [\frac{d}{dx}(\log x + 1) \int 1dx]\, dx$

$\Rightarrow y = x \log (x + 1) - \int \frac{x}{x+1}dx$

$\Rightarrow y = x \log (x + 1) - \int \left(1 - \frac{1}{x+1}\right) dx$

$\Rightarrow y = x \log (x + 1) - x + \log (x + 1) + C$ ...(i)

It is given that y(0) = 3

$\therefore 3 = 0 \times \log (0 + 1) - 0 + \log (0 + 1) + C$

$\Rightarrow C = 3$

Substituting the value of C in (i), we get

$y = x \log (x + 1) + \log (x + 1) - x + 3$

$\Rightarrow y = (x + 1) \log (x + 1) - x + 3$

Hence, $y = (x + 1) \log (x + 1) - x + 3$ is the solution to the given differential equation.

OR

If P denotes the principal at any time t and the rate of interest be r % per annum compounded continuously, then according to the law given in the problem, we get

$\frac{dP}{dt} = \frac{Pr}{100}$

$\Rightarrow \frac{dP}{P} = \frac{r}{100}dt$

$\Rightarrow \int \frac{1}{P}dP = \frac{r}{100} \int dt$

$\Rightarrow \log P = \frac{rt}{100} + C$ ...(i)

Let P_0 be the initial principal i.e. at t = 0, $P = P_0$

Putting $P = P_0$ in (i), we get

$\log P_0 = C$

Putting $C = \log P_0$ in (i), we get

$\log P = \frac{rt}{100} + \log P_0$

$\Rightarrow \log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$ ...(ii)

i. In this case, we have

   $r = 5$, P_0 = ₹ 100 and P = ₹ 200 = $2P_0$

   Substituting these values in (ii), we have

   $\log 2 = \frac{5}{100}t \Rightarrow t = 20 \log_e 2 = 20 \times 0.6931$ years = 13.862 years.

ii. In this case, we have

   P_0 = ₹ 100, P = ₹ 200 = $2P_0$ and t = 10 years.

   Substituting these values in (ii), we get

   $\log 2 = \frac{10r}{100}t \Rightarrow r = 10 \log 2 = 10 \times 0.6931 = 6.931$

   Hence, r = 6.931% per annum.

iii. In this case, we have

   P_0 = ₹ 1000, r = 5 and t = 10

Substituting these values in (ii), we get

$\log\left(\frac{P}{1000}\right) = \frac{5\times10}{100} = \frac{1}{2} = 0.5 \Rightarrow \frac{P}{1000}$ $= e^{0.5} \Rightarrow$ P = 1000 $\times$ 1.648 = 1648

P = ₹ 1648

27. Face value of the bond C = ₹2,000

As the bond is redeemable at ₹105, so redemption price of the bond is 105 % of its face value.

Therefore, redemption value C = 1.05 $\times$ 2,000 = ₹2,100

Nominal rate i_d = 8% or 0.08

So, R = C $\times$ i_d = 2,000 $\times$ 0.08 = ₹160

No. of periods before redemption n = 10

Annual yield rate i = 10 % or 0.1

Therefore, purchase price V is given by,

$V = R\left[\frac{1-(1+i)^{-n}}{i}\right] + C(1+i)^{-n}$

$= 160\left|\frac{1-(1+0.1)^{-10}}{0.1}\right| + 2100(1+0.1)^{-10}$

$= 160\left|\frac{1-(1.1)^{-10}}{0.1}\right| + 2100(1.01)^{-10}$

$= 160\left|\frac{1-0.3855}{0.1}\right| + 2100(0.3855)$

= 982.4 + 809.6

= 1792

Therefore, the present value of the bond is ₹1,792.

28. Let C(x) be the total cost of x units of the product and MC be the marginal cost, then

MC = $\frac{x}{\sqrt{x^2+400}}$ (given)

As MC = $\frac{d}{dx}$ (C(x)), so $\frac{d}{dx}$ (C(x)) = $\frac{x}{\sqrt{x^2+400}}$

$\therefore$ C(x) = $\int \frac{x}{\sqrt{x^2+400}}dx$ (put $\sqrt{x^2+400}$ = t i.e. x^2 + 400 = t^2 $\Rightarrow$ 2x dx = 2t dt i.e. x dx = t dt)

$= \int \frac{tdt}{t} = \int$ 1 dt = t + k, k is constant of integration

$\Rightarrow$ C(x) = $\sqrt{x^2+400}$ + k.

Given fixed cost (in ₹) = 1000 i.e. when x = 0, C(x) = 1000

$\Rightarrow$ 1000 = $\sqrt{0^2+400}$ + k $\Rightarrow$ 1000 = 20 + k $\Rightarrow$ k = 980

$\therefore$ C(x) = $\sqrt{x^2+400}$ + 980

Average cost = $\frac{C(x)}{x} = \frac{\sqrt{x^2+400}}{x} + \frac{980}{x}$.

29. Let X represent the number of winning prizes in 50 lotteries. The trials are Bernoulli trials.

Clearly, X has a binomial distribution with n = 50 and p = $\frac{1}{100}$

$\therefore$ q = 1 - p = 1 $- \frac{1}{100} = \frac{99}{100}$

$\therefore$ P(X = x) = ${}^{n}C_x q^{n-x} p^x = {}^{50}C_x \left(\frac{99}{100}\right)^{50-x} \cdot \left(\frac{1}{100}\right)^x$

i. P (winning at least once) = P(X $\geq$ 1)

= 1 – P (X < 1)

= 1 - P(X = 0)

= 1 - ${}^{50}C_0 \left(\frac{99}{100}\right)^{50}$

= $1 - 1 \cdot \left(\frac{99}{100}\right)^{50}$

= 1 - $\left(\frac{99}{100}\right)^{50}$

ii. P (winning exactly once) = P (X = 1)

= ${}^{50}C_1 \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{1}{100}\right)^1$

= 50 $\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49}$

$= \frac{1}{2}\left(\frac{99}{100}\right)^{49}$

iii. P (at least twice) = P(x $\geq$ 2)

= 1 – P(X < 2)

$= 1 - P(X \leq 1)$
$= 1 - [P(X = 0) + P(X = 1)]$
$= [1 - P(X = 0) - P(X = 1)]$
$= 1 - \left(\frac{99}{100}\right)^{50} - \frac{1}{2} \cdot \left(\frac{99}{100}\right)^{49}$
$= 1 - \left(\frac{99}{100}\right)^{49} \left(\frac{99}{100} + \frac{1}{2}\right]$
$= 1 - \left(\frac{99}{100}\right)^{49} \cdot \left(\frac{149}{100}\right)$
$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49}$

OR

Let X be a normal distribution random variable, then
$P(X < 45) = 31\%$ i.e. 0.31 and $P(X > 64) = 8\%$ i.e. 0.08
Let the mean and the standard deviation of the distribution be μ and σ respectively, then
for X = 45, $Z = \frac{45-\mu}{\sigma}$ and for X = 64, $Z = \frac{64-\mu}{\sigma}$
$\therefore P(X < 45) = P\left(Z < \frac{45-\mu}{\sigma}\right) = 0.31$
$\Rightarrow P\left(Z < \frac{45-\mu}{\sigma}\right) = P(Z < -0.5)$ (using table)
$\Rightarrow \frac{45-\mu}{\sigma} = -0.5$ ...(i)
and $P(X > 64) = P\left(Z > \frac{64-\mu}{\sigma}\right) = 0.08 = 1 - 0.92$
$\Rightarrow P\left(Z > \frac{64-\mu}{\sigma}\right) = 1 - P(Z \leq 1.4) = P(Z > 1.4)$ (using table)
$\Rightarrow \frac{64-\mu}{\sigma} = 1.4$ ...(ii)
Dividing equation (i) by (ii), we get
$\frac{45-\mu}{64-\mu} = \frac{-0.5}{1.4} \Rightarrow 63 - 1.4\mu = -32 + 0.5\mu$
$\Rightarrow 95 = 1.9\mu \Rightarrow \mu = 50$.
Substituting $\mu = 50$ in equation (i), we get
$\frac{45-50}{\sigma} = -0.5 \Rightarrow \sigma = 10$
Hence, mean 50 and standard deviation $= \sigma = 10$

30. We have the following table

Year	No. of failures	5-Yearly Moving Totals	5-Yearly Moving Averages
2003	23	-	-
2004	26	-	-
2005	28	129	25.8
2006	32	118	23.6
2007	20	104	20.8
2008	12	86	17.2
2009	12	63	12.6
2010	10	56	11.2
2011	9	55	11.0
2012	13	57	11.4
2013	11	59	11.8
2014	14	59	11.8
2015	12	49	9.8
2016	9	39	7.8
2017	3	-	-
2018	1	-	-

31. Given μ_0 = 15, n = 50, $\bar{x}$ = 14.15, σ = 3

i. Z = $\frac{\bar{x}-\mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{14.15-15}{\frac{3}{\sqrt{50}}} = \frac{-0.85\times\sqrt{50}}{3}$

= -2.003

$\therefore$ Z = -2

ii. $\because$ Z = -2 < 0

So, p-value = 2(Area under the standard normal curve to the left of Z)

= 2 $\times$ (0.0228) = 0.0456

$\therefore$ p-value = 0.0456

iii. $\because$ p-value < 0.05 (Given α = 0.05)

So, reject H_0

iv. Reject H_0 if Z $\leq -Z_{\frac{\alpha}{2}}$

$\because -Z_{\frac{\alpha}{2}}$ = $-Z_{0.025}$ = -1.96

$\because$ -2 < -1.96

So, reject H_0

Section D

32. Here

D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{vmatrix}$ = 1(10 - 9) - 1(5 - 3) + 1(3 - 2)

= 1 - 2 + 1 = 0

D_1 = $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 7 & 3 & 5 \end{vmatrix}$ = 1(10 - 9) - 1(20 - 21) + 1(12 - 14)

= 1 + 1 - 2 = 0

D_2 = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 7 & 5 \end{vmatrix}$ = 1(20 - 21)-1(5 - 3) + 1(7 - 4)

= -1 - 2 + 3 = 0 and

D_3 = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 7 \end{vmatrix}$ = 1(14 - 12) -1(7 - 4) + 1(3 - 2)

= 2 - 3 + 1 = 0

Thus, D = D_1 = D_2 = D_3 = 0, therefore, the given system may or may not be consistent. Let us solve the first two equations for x and y in terms of z. These equations can be written as:

x + y = 1 - z

x + 2y = 4 - 3z

To solve these equations, we use Cramer's rule.

D = $\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$ = 2 - 1 = 1,

D_1 = $\begin{vmatrix} 1-z & 1 \\ 4-3z & 2 \end{vmatrix}$ = 2 - 2z - 4 + 3z = z - 2, and

D_2 = $\begin{vmatrix} 1 & 1-z \\ 1 & 4-3z \end{vmatrix}$ = 4 - 3z - 1 + z = 3 - 2z

By Cramer's rule,

x = $\frac{D_1}{D} = \frac{z-2}{1}$ = z - 2, y = $\frac{D_2}{D} = \frac{3-2z}{1}$ = 3 - 2z.

Let

z = k where k is arbitrary number, then we get

x = k - 2, y = 3 - 2k, z = k, where k is any number.

Note that these values satisfy the third equation i.e. x + 3y + 5z = 7 of the given system. Hence, the system is consistent and it has infinitely many solutions given by

x = k - 2, y = 3 - 2k, z = k where k is any number.

OR

The given system of equations can be written as AX = B,

where $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 2 \\ 2 & 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$ and $B = \begin{bmatrix} 85 \\ 105 \\ 110 \end{bmatrix}$

Now, |A| = 1(4 - 6)-2(6 - 4) + 3(9 - 4)

= -2 - 4 + 15 = 9 $\neq$ 0

$\Rightarrow$ A^{-1} exists and so the given system has a unique solution

$X = A^{-1} B$

Now, $A_{11} = \begin{vmatrix} 2 & 2 \\ 3 & 2 \end{vmatrix} = -2$, $A_{12} = -\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix} = -2$, $A_{13} = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 5$,

$A_{21} = -\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 5$, $A_{22} = \begin{vmatrix} 1 & 3 \\ 2 & 2 \end{vmatrix} = -4$, $A_{23} = -\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1$,

$\therefore A_{31} = \begin{vmatrix} 2 & 3 \\ 2 & 2 \end{vmatrix} = -2$, $A_{32} = -\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix} = 7$, $A_{33} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = -4$

$\therefore \text{adj } A = \begin{bmatrix} -2 & -2 & 5 \\ 5 & -4 & 1 \\ -2 & 7 & -4 \end{bmatrix} = \begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix}$

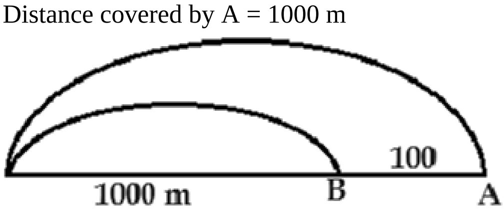
$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{9} \begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix}$

$\therefore X = A^{-1} B = \frac{1}{9} \begin{bmatrix} -2 & 5 & -2 \\ -2 & -4 & 7 \\ 5 & 1 & -4 \end{bmatrix} \begin{bmatrix} 85 \\ 105 \\ 110 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 135 \\ 180 \\ 90 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \\ 10 \end{bmatrix}$

$\therefore p_1 = 15, p_2 = 20, p_3 = 10.$

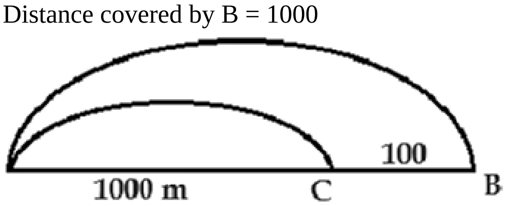
33. Distance covered by A = 1000 m



Distance covered by B = 900 m

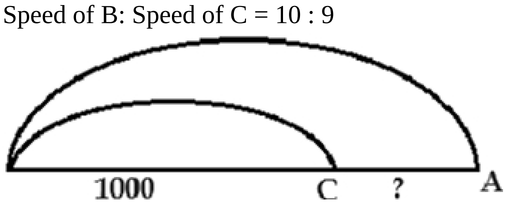
Speed of A: speed of B = 10 : 9

Distance covered by B = 1000



Distance covered by C = 900

Speed of B: Speed of C = 10 : 9



$\therefore$ A : B : C = 100 : 90 : 81

= 1000 : 900 : 81

A : B = 10 : 9

10 : 9.

When A covers 1000 meter C covers 810 metes

$\therefore$ Required distance cover = 1000 - 810

= 190 metre.

34. Let X be a random variable denoting the number of sixes. Since the die is thrown twice, X can take values 0, 1, and 2.

Let S_i denote the event that a six occurs on the die in ith throw and F_i denote the event that the six does not occur in the ith throw.

Then, we have,

P(X = 0) = Probability of not getting six in both the throws

= P(F_1 and F_2)

$= P(F_1 \cap F_2)$

= P(F_1)P(F_2) ... [because F_1, F_2 are independent events]

$= \frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$

P(X = 1) = Probability of getting one six in two throws

= P[(F_1 and S_2) or (S_1 and F_2)]

$= P[(F_1 \cap S_2) \cup (S_1 \cap F_2)]$ ...[By addition Theorem]

$= P(F_1)P(S_2) + P(S_1)P(F_2)$ ...[By multiplication theorem for independent events]

$= \frac{5}{6} \times \frac{1}{6} + \frac{1}{6} \times \frac{5}{6} = \frac{10}{36} = \frac{5}{18}$

and, P(X = 2) = Probability of getting sixes in both the throws

$= P(S_1 \cap S_2)$

= P(S_1) P(S_2)

$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

Therefore, the probability distribution of X is:

X	0	1	2
P(X)	$\frac{25}{36}$	$\frac{5}{18}$	$\frac{1}{36}$

OR

Let p be the probability of selecting a page out of 200 pages. Then,

p = $\frac{1}{200}$ = 0.005

Since p is small, we use the Poisson's distribution

Here, n = Total number of misprints = 220

Average number of misprints in a page= n p

$\Rightarrow$ m = np $\Rightarrow$ m = 220 $\times$ 0.005 = 1.1

Let X denote the number of misprints in a page. Then, X follows Poisson's distribution such that

P(X = r) = $\frac{m^r}{r!}$ e^{-m}, r = 0, 1, 2, 3, ...

$= \frac{(1.1)^r}{r!}$ $e^{1.1}$, r = 0, 1, 2, ...

i. Required probability = P (X = 0)

= $e^{-1.1}$ = 0.33287

ii. Required probability = P (X = 1)

$= \frac{1.1}{1!}$ $e^{-1.1}$ = 1.1 $\times$ 0.33287 = 0.366157

iii. Required probability = P (X = 2)

$= \frac{m^2 e^{-m}}{2!}$

$= \frac{(1.1)^2}{2} \times$ $e^{-1.1}$

$= \frac{1.21 \times 0.33287}{2}$

= 0.20138

iv. Required probability = P (X > 2)

= 1 - P(X < 2)

= 1 - [P(X = 0) + P(X = 1)]

= 1 - [0.33287 + 0.366157] [Using (i) and (ii)]

= 0.300973

35. It is given that C = ₹50,000 and r = 0.08

The depreciation charge for the 8th year is obtained by subtracting the book value at the end of the 8th year from the book value at the end of the 7th year.

The book value at the end of the 7th year.

= C$(1 - r)^7$ = 50,000 $(1 - 0.08)^7$ = 50,000 $(0.92)^7$

= 50,000 (0.5578466)

= ₹27892.33

The book value at the end of the 8th year

= $C(1 - r)^8$ = 50,000 $(1 - 0.08)^8$

= 50000 $(0.92)^8$

= 50,000 (0.5132188)

= ₹25660.94

Hence depreciation charge for the 8th year

= ₹ 27892.33 - ₹ 25660.94

= ₹ 2231.39

The scrap value of the machine is given by (book value at the end of 10th year)

S = $C(1 - r)^{10}$ = 50,000 $(1 - 0.08)^{10}$

= 50,000 $(0.92)^{10}$

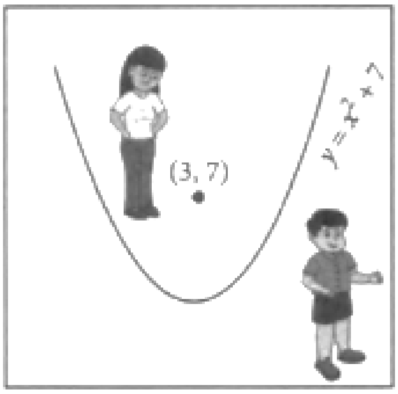
= 50,000 (0.4343884)

= ₹21719.42

Section E

36. **Read the text carefully and answer the questions:**

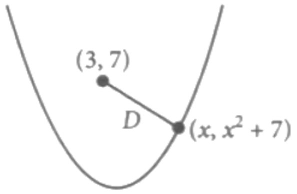
A student Mohit is running on a playground along the curve given by $y = x^2 + 7$. Another student Preeti standing at point (3, 7) on playground wants to hit Mohit by paper ball when Mohit is nearest to Preeti.



(i) For all values of x, $y = x^2 + 7$

$\therefore$ Arun's position at any point of x will be $(x, x^2 + 7)$

(ii) Distance between Mohit and Preeti, i.e., D = $\sqrt{(x-3)^2 + (x^2 + 7 - 7)^2}$



= $\sqrt{(x-3)^2 + x^4}$

(iii) We have, D = $\sqrt{(x-3)^2 + x^4}$

$\therefore D^2 = (x - 3)^2 + x^4$

Now, $\frac{d}{dx}\left(D^2\right) = 2(x - 3) + 4x^3 = 0$

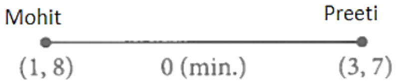
$\Rightarrow 4x^3 + 2x - 6 = 0 \Rightarrow 2x^3 + x - 3 = 0$

$\Rightarrow (x - 1)(2x^2 + 2x + 3) = 0$

$\therefore x = 1$ ($\because 2x^2 + 2x + 3 = 0$ will give imaginary values)

OR

We have, D = $\sqrt{(x-3)^2 + x^4}$



$D'(x) = \frac{2(x-3)+4x^3}{2\sqrt{(x-3)^2+x^4}} = 0$

$\Rightarrow 2x^3 + x - 3 = 0$

$\Rightarrow x = 1$

Clearly, D"(x) at x = 1 is > 0

$\therefore$ Value of x for which D will be minimum is 1 For x = 1, y = 8.

Thus, the required position is (1, 8).

37. **Read the text carefully and answer the questions:**

The nominal rate of return shows the yield of an investment over time without accounting for negative elements such as inflation or taxes. By calculating the nominal rate of return, you can compare the performance of your assets easily, regardless of the inflation rate or differing spans of time for each investment. By obtaining a bird's-eye view of how your assets are growing, you can make more prudent investment decisions in the future.

(i) 12.5%

(ii) $11\frac{7}{13}\%$

(iii)12%

OR

₹70

38. i. Since, Aeroplane can carry a maximum of 200 passengers

$\therefore x + y \leq 200$

ii. Since, Airline reserves at least 20 seats for executive class

$\Rightarrow x \leq 200$

Also atleast four times as many passengers prefer to travel by economy class than by executive class.

$\Rightarrow y = 4x$

$\Rightarrow x + 4x \leq 200$ [$\because x + y \leq 200$]

$\Rightarrow 5x \leq 200 \Rightarrow x \leq 40$

$\Rightarrow x \geq 20$ and $x \leq 40$

iii. Profit on executive class = 400x

Profit on executive class = 300y

$\therefore$ Total profit Z = 400x + 300y

iv. We have

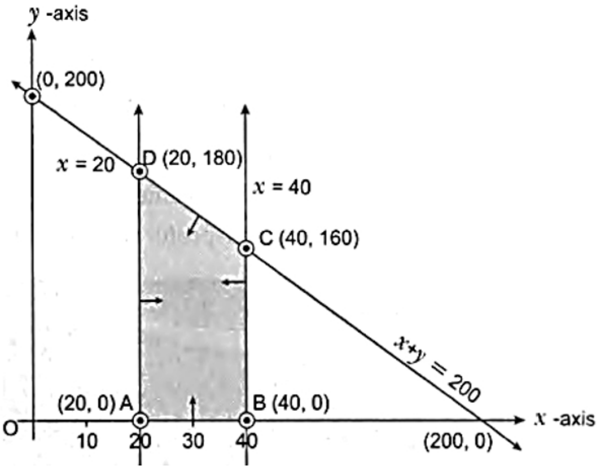
Z = 400x + 300y which is to be maximise under constraints

$x + y \leq 200$

$x \leq 40$

$x \geq 20$, $y \geq 0$

Here, ABCD in bounded feasible region with comer points A(20, 0), B(40, 0), C(40,160), D (20,180).



Now we evaluate Z at each corner points.

Corner point	Z = 400x + 300y
A(20, 0)	8000
B(40, 0)	16000

C(40, 160)	64000 $\leftarrow$ Maximum
D(20, 180)	62000

For maximum profit x = 40, y = 160

v. We have

Z = 400x + 300y

= 400 $\times$ 40 + 300 $\times$ 160

= 16000 + 48000

= ₹ 64000 [$\because$ For maximum profit x = 40, y = 160]

OR

i. The point A(5, 10) lies on the equation y - 2x = 0, therefore the equation of line OA is y - 2x = 0.
ii. Point on line BC i.e., C(0, 2) lies on the equation y - 2x = 2, therefore equation of line BC is y - 2x = 2.
iii. Point B is the intersection point of line BC and BD.

So, substituting x = 5 in y - 2x = 2,

we get y = 12

Thus, required coordinates are (5, 12).

OR

The required constraints for L.P.P. are

$y \geq 2x$

$y - 2x \leq 2$

$x \leq 5$

$x \geq 0, y \geq 0$